Energy conversion of rocks in process of unloading confining pressure under different unloading paths

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Abstract: Based on energy theory and tests of rocks with initial confining pressures of 10, 20 and 30 MPa under different unloading paths, the processes of strain energy conversion were investigated. The absorbing strain energy for axial compression, the dissipating strain energy for plastic deformation and cracks propagation, the expending strain energy for circumferential deformation, and the storing and releasing elastic strain energy were considered. Unloading paths included the condition of fixing axial pressure and unloading axial pressure, increasing axial pressure and unloading confining pressure, as well as unloading axial pressure and confining pressure simultaneously. Results show that expending strain energy for circumferential deformation has mainly evolved from absorbing strain energy for axial compression in three unloading paths during unloading processes. Dissipating strain energy is significantly increased only near the peak point. The effect of initial confining pressure on strain energy is significantly higher than that of unloading path. The strain energy is linearly increased with increasing initial confining pressure. The unloading path and initial confining pressure also have great influence on the energy dissipation. The conversion rate of strain energy in three paths is increased with increasing initial confining pressure, and the effect of initial confining pressure on conversion rate of strain energy is related with the unloading paths.

Key words: unloading paths; axial pressure; confining pressure; strain energy; energy conversion

1 Introduction

Underground stope excavation results in unbalancing of rock stress. With continuous propulsion of mining face, the exposure area of surrounding rock is increasing constantly [1–3]. It causes sudden release of the elastic strain energy of rocks, which would lead to severe rock failure, such as rock burst, spalling, collapse and other geological disasters. Therefore, based on energy theory, the mechanical behavior of rock during loading has been widely investigated by means of laboratory experiment and numerical modeling, for example, energy feature under uniaxial compression, release of energy, and the relationship of deformation, confining pressure and energy [4–11].

However, investigations indicated that rock behavior under unloading is different from that under loading [12–17]. In order to better understand the mechanical properties of unloading, many experts and scholars have made great efforts on the research of this issue, and have obtained many worthy achievements, such as, deformation modulus of rock mass under different unloading rates; energy dissipation of rock in the process of unloading confining pressure [18–29]. But in the previous studies, different initial stresses and different unloadings path were not considered together to research the release and dissipation of energy in unloading.

The specimens were studied in the present study by means of conventional tri-axial unloading tests at different unloading paths and initial confining pressures. The strain energy conversion was researched, including absorbing strain energy for axial compression, dissipating strain energy for plastic deformation and cracks propagation, expending strain energy for circumferential deformation, storing and releasing elastic strain energy.

2 Experimental

2.1 Test conditions

The study was conducted at the Geo-Mechanical Test Center, Central South University, on the MTS815
compression machine, under strict test conditions incorporated into the test program. The tester was a rigid machine that was produced by MTS Co. Ltd. (USA) specifically for versatile rock testing. This machine incorporated electro-hydraulic servo control with automatic pressure relief, as well as axis servo control and the measurement system. 

The sample was a gray colored granitic rock with dimensions of $d=50 \text{ mm} \times 100 \text{ mm}$. 39 specimens had a longitudinal wave velocity of $3200-3800 \text{ m/s}$, a density of $2.5 \text{ g/cm}^3$ (when measured in natural water), and a uni-axial compressive strength of 80 MPa. And every test was conducted for 3 times.

### 2.2 Test program

The specimens were put into three groups for testing under different initial conditions as listed in Table 1. The axial pressure which is 80% of the tri-axial strength was applied in a strain-controlled way. The initial confining pressure was set to be three levels, 10, 20 and 30 MPa, respectively.

The detailed test procedures are as follows.

Group I: maintaining a constant axial pressure while decreasing confining pressure.

Group II: increasing axial pressure while decreasing confining pressure.

Group III: decreasing axial pressure and confining pressure simultaneously.

#### Table 1 Initial geostress condition of various rock samples

<table>
<thead>
<tr>
<th>Sample number</th>
<th>Confining pressure/MPa</th>
<th>Tri-axial strength/MPa</th>
<th>Axial pressure/MPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1, X1-1, X1-2, X1-3</td>
<td>10</td>
<td>237.5</td>
<td>190</td>
</tr>
<tr>
<td>S2, X2-1, X2-2, X2-3</td>
<td>20</td>
<td>306.5</td>
<td>245</td>
</tr>
<tr>
<td>S3, X3-1, X3-2, X3-3</td>
<td>30</td>
<td>362.5</td>
<td>290</td>
</tr>
</tbody>
</table>

### 3 Energy evolutions in process of unloading

#### 3.1 Calculation method of strain energy

In the tri-axial tests of rocks, test machine does positive work to the specimen in the axial direction; confining pressure does negative work, due to the radial dilation of the specimen. So the strain energy $U$ of rock in the whole process of tri-axial tests can be expressed as

$$U = U_1 + U_3$$  \hspace{1cm} (1)

where $U_1$ is the absorbed strain energy due to axial compression by $\sigma_1$; $U_3$ is the consumed strain energy by negative work of $\sigma_3$.

The total strain energy $U$ can be divided into two parts, the elastic strain energy $U_e$ which is stored in the specimen, and the plasto-damage strain energy $U_d$ which is responsible for plastic deformation and crack propagation in rock specimen. That is

$$U = U_e + U_d$$  \hspace{1cm} (2)

The absorbed and consumed strain energies ($U_1$ and $U_3$) at any time during the test can be obtained by integral of the stress–strain curve as follows:

$$U_1 = \int_0^\tau \varepsilon_1^1 \sigma_1 d\varepsilon_1$$  \hspace{1cm} (3)

$$U_3 = \int_0^\tau \varepsilon_3^1 \sigma_3 d\varepsilon_3$$  \hspace{1cm} (4)

where $\varepsilon_1^1$ and $\varepsilon_3^1$ are the axial and radial strain at time $\tau$, respectively.

The integral in Eqs. (3) and (4) are normally calculated in practice through area summation of tiny trapezoids, according to the definition of integral calculation. That is

$$U_1 = \sum_{i=1}^n \left( \frac{1}{2} (\sigma_1^i + \sigma_1^{i+1}) (\varepsilon_1^{i+1} - \varepsilon_1^i) \right)$$  \hspace{1cm} (5)

$$U_3 = \sum_{i=1}^n (\sigma_3^i + \sigma_3^{i+1}) (\varepsilon_3^{i+1} - \varepsilon_3^i)$$  \hspace{1cm} (6)

where $n$ is the total number of trapezoids of stress–strain curve, and $i$ is the segmentation points.

The elastic strain energy $U_e$ at any time under the condition of conventional tri-axial compression can be obtained as follows:

$$U_e = \frac{1}{2E_u} \left[ \varepsilon_1^2 + 2\varepsilon_3^2 - 2\mu_u' (2\sigma_1 \varepsilon_3 + \sigma_3^2) \right]$$  \hspace{1cm} (7)

where $E_u'$ and $\mu_u'$ are the unloading modulus of elasticity and Poisson ratio at time $\tau$, respectively.

#### 3.2 Evolution way of strain energy in process of unloading

##### 3.2.1 Group I

Figure 1 presents the stress–strain curves of Group I. As shown in Fig. 1, the axial strain increases from 0.0045 to 0.007, and the lateral strain increases from $-0.001$ to $-0.011$. So lateral strain increases sharply, which is 3–5 times quicker than that of the axial strain.

Energy conversion chart of Group I is shown in Fig. 2. In the process of unloading confining pressure, because the axial pressure keeps constant, the elastic strain energy $U_{e1}$ which is the enclosed area by purple lines is equal to the elastic strain energy $U_{e2}$ which is the enclosed area by black lines. Test machine does positive work to specimens in the axial direction. Confining pressure does negative work in lateral direction in process of unloading. So dissipated strain energy of
specimens in Group I can be obtained as follows:

\[ U_{di} = U_{el} + 2 \int_{\varepsilon_{1}}^{\varepsilon_{2}} \sigma_{3} \varepsilon_{3} \, d\varepsilon_{3} = \int_{\varepsilon_{1}}^{\varepsilon_{2}} \sigma_{1} \varepsilon_{1} \, d\varepsilon_{1} + 2 \int_{\varepsilon_{1}}^{\varepsilon_{2}} \sigma_{3} \varepsilon_{3} \, d\varepsilon_{3} \]  

where \( U_{di} \) is the area enclosed by purple dotted lines and black dashed lines in Fig. 2.

3.2.2 Group II

Figure 3 presents the stress–strain curves of Group II. As shown in Fig. 3, the specimens show obvious lateral expansion which is more obvious than that in Group I in the process of unloading. This is mainly because more energy is provided by increasing axial compression to specimens, and the rock sample is broken without enough time to dilate.

Energy conversion chart of Group II is shown in Fig. 4. The elastic strain energy \( U_{el} \) which is the enclosed area by purple lines is less than the elastic strain energy \( U_{el} \) which is enclosed area by black lines. It illustrates that rocks absorb the elastic strain in the process of unloading. So dissipating strain energy of specimens in Group II can be obtained as follows:

\[ U_{di} = U_{el} - U_{el} + U_{el} + 2 \int_{\varepsilon_{1}}^{\varepsilon_{2}} \sigma_{3} \varepsilon_{3} \, d\varepsilon_{3} \]  

where \( U_{di} \) is the area enclosed by purple dotted lines and black dashed lines in Fig. 4.

3.2.3 Group III

Figure 5 presents the stress–strain curves of Group III. The lateral strain increases rapidly and axial strain increases slowly, their variation is more than that in Group I. This is because unloading in axial is equivalent to an increase of tensile stress in the lateral direction. With the development of tensile stress, crack develops rapidly from the exterior to internal. So it shows obvious lateral expansion, which would lead to the damage of specimens eventually.

Energy conversion chart of Group III is shown in Fig. 6. As shown in Fig. 6, the elastic strain energy \( U_{el} \) which is the enclosed area by purple lines is more than the elastic strain energy \( U_{el} \) which is enclosed area by black lines. It illustrates that rocks release the elastic strain in the process of unloading. So dissipating strain energy of specimens in Group III can be obtained as follows:

\[ U_{di} = U_{el} - U_{el} + U_{el} + 2 \int_{\varepsilon_{1}}^{\varepsilon_{2}} \sigma_{3} \varepsilon_{3} \, d\varepsilon_{3} \]  

where \( U_{di} \) is the area enclosed by purple dotted lines.
3.3 Analyses of evolution process of strain energy

Figure 7 presents the time history curves of strain energy accumulation, dissipation and release for specimens tested in three unloading paths. Elastic strain energy in Group I remains unchanged in the process of unloading. So the curves of elastic strain energy in Group I have not been mapped. The results can be summarized as follows.

1) In the initial stage of unloading confining pressure, absorbed strain energy $U_1$ for axial compression rapidly increases, while the dissipation energy $U_d$ remains unchanged. It shows that the specimens stay in the elastic stage. The absorbed strain energy $U'_1$ is mainly converted into the expending strain energy $U'_3$ for circumferential deformation in Group I; In Group II, the absorbed strain energy $U'_1$ is mainly converted into the expending strain energy $U''_3$ for circumferential deformation and elastic strain energy $U''_e$ which is stored in specimens; In Group III, because the elastic strain energy $U''_e$ releases very few in the stage of unloading, the absorbed strain energy $U''_1$ is mainly converted into the expending strain energy $U''_3$ for circumferential deformation.

2) Near the peak point, the dissipation energy $U_d$ in three groups increases quickly. This indicates that the specimen has damaged plastic deformation and crack extension. And $U''_e$ to be stored in the specimen in Group II reaches the highest value near the peak point. When the specimen is damaged, the elastic strain energy in specimen rapidly releases. But after the peak point, there is $U'_1 + U''_e - U''_3 - U_d > 0$, the residual energy may account for the rock burst, which happens sometime during unloading by excavation. And there is $0 < U''_1 + U''_e - U''_3 - U_d < U'_1 + U''_e - U'_3 - U_d$ , so the possibility of rock burst in Group III is less than that in
Table 2 presents average strain energy at peak point during unloading. The can be summarized as follows.

1) At the peak point, \( U_1, U_d, U_3, U_e \) are all increased, positively or negatively, with the increase in confining pressure. And at the peak point, there is \( U'_1 > U'_d > U'_3 \), so when the specimens are damaged, the possibility of rock burst in Group II is maximum.

2) Under the high initial confining pressure (30 MPa), there is \( U'''_3 > U''_1 > U'_1 \), which shows that the expansion of specimens in Group III is maximum, and that in Group II is minimum.

3) The initial confining pressure has much more influence on \( U_1, U_3 \) and \( U_e \), which is nearly linearly increased with initial confining pressure before peak point. This indicates that the pre-unloading stress state has much influence on accumulation of \( U_e \). And the unloading path and initial confining pressure both have much influence on \( U_d \).

### Table 2

<table>
<thead>
<tr>
<th>Initial confining pressure/MPa</th>
<th>( U_1 )/MJ·m(^{-3})</th>
<th>( U_3 )/MJ·m(^{-3})</th>
<th>( U_e )/MJ·m(^{-3})</th>
<th>( U_d )/MJ·m(^{-3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>I 10</td>
<td>0.100</td>
<td>-0.068</td>
<td>0.416</td>
<td>0.032</td>
</tr>
<tr>
<td>20</td>
<td>0.150</td>
<td>-0.104</td>
<td>0.642</td>
<td>0.047</td>
</tr>
<tr>
<td>30</td>
<td>0.247</td>
<td>-0.189</td>
<td>0.942</td>
<td>0.058</td>
</tr>
<tr>
<td>II 10</td>
<td>0.112</td>
<td>-0.068</td>
<td>0.446</td>
<td>0.019</td>
</tr>
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<td>0.267</td>
<td>-0.169</td>
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<tr>
<td>III 10</td>
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<td>-0.043</td>
<td>0.403</td>
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<td>30</td>
<td>0.165</td>
<td>-0.219</td>
<td>0.843</td>
<td>0.051</td>
</tr>
</tbody>
</table>

### 3.4 Conversion rate

The increment of stain energy \( \Delta U \) from the initial point of unloading to the peak point is divided by the elapsed time \( \Delta t \) to get the pre-peak conversion rate \( u \) of strain energy [24–26], that is

\[
u = \frac{\Delta U}{\Delta t}
\]

Figure 8 presents the curve of pre-peak conversion rate of strain energy versus initial confining pressure, for \( U_3, U_e \) and \( U_d \) (\( U_3 \) and \( U_e \) released take the absolute values).

1) Under low initial confining pressure (10 MPa), there is more difference about the conversion rates of expending strain energy \( U_3 \) in three groups. The conversion rate of \( U'_3 \) in Group I is about two times faster than that of \( U'_3 \) in Group III. But under high initial confining pressure (30 MPa), the difference about the conversion rates of expending strain energy \( U_3 \) in three groups is not obvious. The conversion rates of \( U'_3 \) and that of \( U'''_3 \) are linearly increased with increasing initial confining pressure. However, the conversion rates of \( U'''_3 \) is faster than that of \( U'_3 \) and \( U''_3 \). This indicates that the expansion in Group III is most obvious.

2) The conversion rate of \( U_d \) is increased with initial confining pressure. And under the same confining
pressure, the conversion rate of $U_0''$ in Group III is minimum. Because in the process of unloading path which is decreasing axial pressure and confining pressure, the time is longer, and the absorbed strain energy $U_1''$ is mainly consumed by expanding strain energy $U_1'$.  

3) The absorption rate of $U_2''$ in Group II and the release rate of $U_2''$ in Group III are increased with initial confining pressure. However, when the initial confining pressure is greater than 20 MPa, the absorption rate of $U_2''$ in Group II is increased obviously, indicating a sudden failure of the specimen. This is because the great amount of $U_2''$ has been stored in the specimen before unloading. This observation may explain severe rock burst under high geostress and quick unloading.

4) The pre-peak conversion rate of $U_3$ is faster than that of $U_4$ and $U_c$. This may indicate that the absorbed strain energy $U_1$ is mainly consumed by expanding strain energy $U_3$, while the dissipating strain energy $U_4$ takes the least. Near the peak point, the dissipating strain energy $U_0$ increases faster because of the plastic deformation and crack propagation.

4 Conclusions

1) Expending strain energy for circumferential deformation has mainly evolved from absorbing strain energy for axial compression in three unloading paths, and expansion in Group III is more obvious than that in Group I and Group II.

2) The influence of initial confining pressure is more significant than the unloading paths on $U_1$, $U_2$, and $U_c$, which is increased with the larger value of initial confining pressure.

3) In Group II, there is $U_1'' + U_2'' - U_3'' - U_4'' > 0$ near the peak point, the residual energy may account for the rock burst, which happens sometimes during unloading by excavation. And there is $0 < U_1'' + U_2'' - U_3'' - U_4'' < U_1' + U_2' - U_3' - U_4'$, so the possibility of rock burst in Group III is less than that in Group II.

4) The conversion rates of $U_3$, $U_0$, $U_c$ are increased with increasing initial confining pressure. And this relationship is related with the unloading paths.

References


不同卸荷路径下岩石卸荷破坏的能量演化规律

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摘 要: 基于岩石能量交换原理和 3 种不同卸荷路径下(恒轴压卸围压、加轴压卸围压、轴压围压同时卸载)卸围压(初始围压为 10 MPa、20 MPa、30 MPa)试验, 研究卸荷条件下岩石轴向吸收应变能、环向扩容消耗应变能、弹性应变能以及耗散能的演化特征与演化速率。研究结果表明, 3 个方案中, 岩石轴向吸收的应变能主要转化为环向扩容消耗应变能, 扩容程度为: 方案 3>方案 1>方案 2, 而转化为耗散能较少, 只有在临近破坏时耗散能才明显增加。初始围压对轴向应变能、环向扩容消耗应变能及弹性应变能的影响程度明显大于卸载路径, 且都随着初始围压的增大呈近似线性增加。卸载路径和初始围压对耗散能有显著的影响。三个方案中应变能的演化速率均随着初始围压的增大而增加, 轴向围压对轴向应变能演化速率的影响与卸载路径有关。

关键词: 卸荷路径; 轴压; 围压; 应变能; 能量演化

(Edited by Yun-bin HE)